

Thomas Mikosch

Non-Life Insurance Mathematics

An Introduction with
Stochastic Processes



Springer

Contents

Guidelines to the Reader	1
--------------------------------	---

Part I Collective Risk Models

1 The Basic Model	7
2 Models for the Claim Number Process	13
2.1 The Poisson Process	13
2.1.1 The Homogeneous Poisson Process, the Intensity Function, the Cramér-Lundberg Model	15
2.1.2 The Markov Property	18
2.1.3 Relations Between the Homogeneous and the Inhomogeneous Poisson Process	20
2.1.4 The Homogeneous Poisson Process as a Renewal Process	21
2.1.5 The Distribution of the Inter-Arrival Times	26
2.1.6 The Order Statistics Property	28
2.1.7 A Discussion of the Arrival Times of the Danish Fire Insurance Data 1980-1990	38
2.1.8 An Informal Discussion of Transformed and Generalized Poisson Processes	41
Exercises	52
2.2 The Renewal Process	59
2.2.1 Basic Properties	59
2.2.2 An Informal Discussion of Renewal Theory	66
Exercises	71
2.3 The Mixed Poisson Process	71
Exercises	75

X Contents

3	The Total Claim Amount	77	
3.1	The Order of Magnitude of the Total Claim Amount	78	I
3.1.1	The Mean and the Variance in the Renewal Model	79	-
3.1.2	The Asymptotic Behavior in the Renewal Model	80	5
3.1.3	Classical Premium Calculation Principles	84	
	Exercises	86	
3.2	Claim Size Distributions	88	
3.2.1	An Exploratory Statistical Analysis: QQ-Plots	88	
3.2.2	A Preliminary Discussion of Heavy- and Light-Tailed Distributions	92	6
3.2.3	An Exploratory Statistical Analysis: Mean Excess Plots	94	
3.2.4	Standard Claim Size Distributions and Their Properties	100	
3.2.5	Regularly Varying Claim Sizes and Their Aggregation . . .	105	
3.2.6	Subexponential Distributions	109	
	Exercises	112	
3.3	The Distribution of the Total Claim Amount	115	F
3.3.1	Mixture Distributions	115	I
3.3.2	Space-Time Decomposition of a Compound Poisson Process	121	L
3.3.3	An Exact Numerical Procedure for Calculating the Total Claim Amount Distribution	126	
3.3.4	Approximation to the Distribution of the Total Claim Amount Using the Central Limit Theorem	131	
3.3.5	Approximation to the Distribution of the Total Claim Amount by Monte Carlo Techniques	135	
	Exercises	143	
3.4	Reinsurance Treaties	147	
	Exercises	154	
4	Ruin Theory	155	
4.1	Risk Process, Ruin Probability and Net Profit Condition . . .	155	
	Exercises	160	
4.2	Bounds for the Ruin Probability	161	
4.2.1	Lundberg's Inequality	161	
4.2.2	Exact Asymptotics for the Ruin Probability: the Small Claim Case	166	
4.2.3	The Representation of the Ruin Probability as a Compound Geometric Probability	176	
4.2.4	Exact Asymptotics for the Ruin Probability: the Large Claim Case	178	
	Exercises	181	

Part II Experience Rating

5 Bayes Estimation	191
5.1 The Heterogeneity Model	191
5.2 Bayes Estimation in the Heterogeneity Model	193
Exercises	199
6 Linear Bayes Estimation	203
6.1 An Excursion to Minimum Linear Risk Estimation	204
6.2 The Bühlmann Model.....	208
6.3 Linear Bayes Estimation in the Bühlmann Model	210
6.4 The Bühlmann-Straub Model	213
Exercises	215
References	217
Index	223
List of Abbreviations and Symbols.....	233