

Statistical Inference

Second Edition

George Casella

University of Florida

Roger L. Berger

North Carolina State University

CENGAGE
Learning"

Contents

1	Probability Theory	1
1.1	Set Theory	1
1.2	Basics of Probability Theory	5
1.2.1	Axiomatic Foundations	5
1.2.2	The Calculus of Probabilities	9
1.2.3	Counting	Hi
1.2.4	Enumerating Outcomes	l(i)
1.3	Conditional Probability and Independence	20
1.4	Random Variables	27
1.5	Distribution Functions	29
1.6	Density and Mass Functions	34
1.7	Exercises	37
1.8	Miscellanea	44
2	Transformations and Expectations	47
2.1	Distributions of Functions of a Random Variable	47
2.2	Expected Values	55
2.3	Moments and Moment Generating Functions	59
2.4	Differentiating Under an Integral Sign	68
2.5	Exercises	76
2.6	Miscellanea	82
3	Common Families of Distributions	85
3.1	Introduction	85
3.2	Discrete Distributions	85
3.3	Continuous Distributions	98
3.4	Exponential Families	111
3.5	Location and Scale Families	116

3.6	Inequalities and Identities	121
3.6.1	Probability Inequalities	122
3.6.2	Identities	123
3.7	Exercises	127
3.8	Miscellanea	135
4	Multiple Random Variables	139
4.1	Joint and Marginal Distributions	139
4.2	Conditional Distributions and Independence	147
4.3	Bivariate Transformations	156
4.4	Hierarchical Models and Mixture Distributions	162
4.5	Covariance and Correlation	169
4.6	Multivariate Distributions	177
4.7	Inequalities	186
4.7.1	Numerical Inequalities	186
4.7.2	Functioned Inequalities	189
4.8	Exercises	192
4.9	Miscellanea	2J3
5	Properties of a Random Sample	207
5.1	Basic Concepts of Random Samples	207
5.2	Sums of Random Variables from a Random Sample	211
5.3	Sampling from the Normal Distribution	218
5.3.1	Properties of the Sample Mean and Variance	218
5.3.2	The Derived Distributions: Student's t and Snedecor's F	222
5.4	Order Statistics	226
5.5	Convergence Concepts	232
5.5.1	Convergence in Probability	232
5.5.2	Almost Sure Convergence	234
5.5.3	Convergence in Distribution	235
5.5.4	The Delta Method	240
5.6	Generating a Random Sample	245
5.6.1	Direct Methods	247
5.6.2	Indirect Methods	251
5.6.3	The Accept/Reject Algorithm	253
5.7	Exercises	255
5.8	Miscellanea	267
6	Principles of Data Reduction	271
6.1	Introduction	271
6.2	The Sufficiency Principle	272
6.2.1	Sufficient Statistics	272
6.2.2	Minimal Sufficient Statistics	279
6.2.3	Ancillary Statistics	282
6.2.4	Sufficient, Ancillary, and Complete Statistics	284

6.3	The Likelihood Principle	290
6.3.1	The Likelihood Function	290
6.3.2	The Formal Likelihood Principle	292
6.4	The Equivariance Principle	296
6.5	Exercises	300
6.6	Miscellanea	307
7	Point Estimation	311
7.1	Introduction	311
7.2	Methods of Finding Estimators	312
7.2.1	Method of Moments	312
7.2.2	Maximum Likelihood Estimators	315
7.2.3	Bayes Estimators	324
7.2.4	The EM Algorithm	326
7.3	Methods of Evaluating Estimators	330
7.3.1	Mean Squared Error	330
7.3.2	Best Unbiased Estimators	334
7.3.3	Sufficiency and Unbiasedness	342
7.3.4	Loss Function Optimality	348
7.4	Exercises	355
7.5	Miscellanea	367
8	Hypothesis Testing	373
8.1	Introduction	373
8.2	Methods of Finding Tests	374
8.2.1	Likelihood Ratio Tests	374
8.2.2	Bayesian Tests	379
8.2.3	Union-Intersection and Intersection-Union Tests	380
8.3	Methods of Evaluating Tests	382
8.3.1	Error Probabilities and the Power Function	382
8.3.2	Most Powerful Tests	387
8.3.3	Sizes of Union-Intersection and Intersection-Union Tests	394
8.3.4	p-Values	397
8.3.5	Loss Function Optimality	400
8.4	Exercises	402
8.5	Miscellanea	413
9	Interval Estimation	417
9.1	Introduction	417
9.2	Methods of Finding Interval Estimators	420
9.2.1	Inverting a Test Statistic	420
9.2.2	Pivotal Quantities	427
9.2.3	Pivoting the CDF ,	430
9.2.4	Bayesian Intervals	435

9.3	Methods of Evaluating Interval Estimators	440
9.3.1	Size and Coverage Probability	440
9.3.2	Test-Related Optimality	444
9.3.3	Bayesian Optimality	447
9.3.4	Loss Function Optimality	449
9.4	Exercises	451
9.5	Miscellanea	463
10	Asymptotic Evaluations	467
10.1	Point Estimation-	467
10.1.1	Consistency	467
10.1.2	Efficiency	470
10.1.3	Calculations and Comparisons	473
10.1.4	Bootstrap Standard Errors	478
10.2	Robustness	481
10.2.1	The Mean and the Median	482
10.2.2	M-Estimators	484
10.3	Hypothesis Testing	488
10.3.1	Asymptotic Distribution of LRTs	488
10.3.2	Other Large-Sample Tests	492
10.4	Interval Estimation	496
10.4.1	Approximate Maximum Likelihood Intervals	496
10.4.2	Other Large-Sample Intervals	499
10.5	Exercises	504
10.6	Miscellanea	515
11	Analysis of Variance and Regression	521
11.1	Introduction	521
11.2	Oneway Analysis of Variance	522
11.2.1	Model and Distribution Assumptions	524
11.2.2	The Classic ANOVA Hypothesis	525
11.2.3	Inferences Regarding Linear Combinations of Means	527
11.2.4	The ANOVA F Test	530
11.2.5	Simultaneous Estimation of Contrasts	534
11.2.6	Partitioning Sums of Squares	536
11.3	Simple Linear Regression	539
11.3.1	Least Squares: A Mathematical Solution	542
11.3.2	Best Linear Unbiased Estimators: A Statistical Solution	544
11.3.3	Models and Distribution Assumptions	548
11.3.4	Estimation and Testing with Normal Errors	550
11.3.5	Estimation and Prediction at a Specified $x = x_0$	557
11.3.6	Simultaneous Estimation and Confidence Bands	559
11.4	Exercises	563
11.5	Miscellanea	572

12 Regression Models	577
12.1 Introduction	577
12.2 Regression with Errors in Variables	577
12.2.1 Functional and Structural Relationships	579
12.2.2 A Least Squares Solution	581
12.2.3 Maximum Likelihood Estimation	583
12.2.4 Confidence Sets	588
12.3 Logistic Regression	591
12.3.1 The Model	591
12.3.2 Estimation	593
12.4 Robust Regression	597
12.5 Exercises	602
12.6 Miscellanea	608
Appendix: Computer Algebra	613
Table of Common Distributions	621
References	629
Author Index	645
Subject Index	649