

Annamaria Olivieri • Ermanno Pitacco

Introduction to Insurance Mathematics

Technical and Financial Features
of Risk Transfers

4y Springer

Contents

Risks and insurance	1
1.1 Introduction	1
1.2 "Risk": looking for definitions	1
1.2.1 Some preliminary ideas	1
1.2.2 Transactions with random results	2
1.2.3 A very basic insurable risk	4
1.2.4 Random number of events and random amounts	4
1.2.5 Risks inherent in the individual lifetime	9
1.3 Managing risks	14
1.3.1 General aspects	14
1.3.2 Risk identification and risk assessment	15
1.3.3 Risk management actions	16
1.3.4 Self-insurance versus insurance	18
1.3.5 Monitoring and the Risk Management cycle	21
1.4 Quantifying risks: some models	21
1.4.1 Some preliminary ideas	21
1.4.2 A very basic model	22
1.4.3 Random number of events and random amounts	23
1.4.4 Random sums: a critical assumption	28
1.4.5 Introducing time into valuations	29
1.4.6 Comparing random yields	31
1.4.7 Risk-adjusted valuations	33
1.5 Risk measures	37
1.5.1 Some preliminary ideas	37
1.5.2 Traditional risk measures	38
1.5.3 Downside risk measures	39
1.5.4 Risk measures and capital requirements	40
1.6 Transferring risks	43
1.6.1 Building up a pool	43
1.6.2 Financing the pool	46
1.6.3 The role of the insurer	51

1.7	Insurance products	53
1.7.1	The insurance cover. Policy conditions	53
1.7.2	Some examples	54
1.7.3	Pricing insurance products	56
1.7.4	Premium calculation	57
1.7.5	Technical bases	64
1.7.6	Reserving	66
1.8	References and suggestions for further reading	67
Managing a portfolio of risks		69
2.1	Introduction	69
2.2	Rating: the basics	70
2.2.1	Some preliminary ideas	70
2.2.2	Homogeneous risks	70
2.2.3	Non-homogeneous risks	71
2.2.4	A more general rating system	72
2.2.5	Rating systems and technical equilibrium	73
2.2.6	From risk factors to rating classes	75
2.2.7	Cross-subsidy: mutuality and solidarity	77
2.3	Facing portfolio riskiness	78
2.3.1	Expected outgo versus actual outgo	79
2.3.2	Risk assessment	80
2.3.3	The risk index	82
2.3.4	The probability distribution of the total payment	84
2.3.5	The safety loading	87
2.3.6	Capital allocation and beyond	91
2.3.7	Solvency	94
2.3.8	Creating value	95
2.3.9	Risk management and risk analysis: some remarks	97
2.3.10	The "uncertainty risk"	98
2.4	Reinsurance: the basics	101
2.4.1	General aspects	101
2.4.2	Stop-loss reinsurance	103
2.4.3	From portfolios to contracts	105
2.4.4	Two reinsurance arrangements	107
2.4.5	Examples	109
2.4.6	Optimal reinsurance policy	111
2.5	Reinsurance: further aspects	113
2.5.1	Reinsurance arrangements	113
2.5.2	Random claim sizes. XL reinsurance	114
2.5.3	Catastrophe reinsurance	116
2.5.4	Purposes of reinsurance	118
2.5.5	Insurance-reinsurance networks	119
2.5.6	Reinsurance treaties. Reinsurance programmes	120
2.6	Alternative risk transfers	123

Contents

2.6.1	Some preliminary ideas.	123
2.6.2	Securitization and the role of capital markets.	124
2.6.3	An example: the mortality bonds.	126
2.7	The time dimension.	128
2.7.1	General aspects.	128
2.7.2	Premiums, payments, portfolio fund.	130
2.7.3	Solvency and capital requirements.	131
2.7.4	Generalizing the model.	134
2.7.5	Solvency and capital flows.	135
2.8	References and suggestions for further reading.	137
2.A	Appendix.	138
Life insurance: modeling the lifetime.		141
3.1	Introduction.	141
3.2	Life tables.	141
3.2.1	Elements of a life table.	141
3.2.2	Cohort tables and period tables.	142
3.2.3	Construction of a period life table.	145
3.2.4	"Population" tables versus "market" tables.	147
3.2.5	The life table as a probabilistic model.	148
3.2.6	One-year measures of mortality.	149
3.2.7	A more formal setting: the random lifetime.	153
3.3	Summarizing a life table.	154
3.3.1	The life expectancy.	154
3.3.2	Other markers.	156
3.4	A mortality "law".	156
3.4.1	From tables to parameters.	156
3.4.2	The Heligman-Pollard law.	157
3.5	From the basic model to more general models.	159
3.6	Heterogeneity.	159
3.6.1	Some preliminary ideas.	159
3.6.2	Rating classes.	161
3.6.3	Substandard risks.	163
3.6.4	The "factor formula".	164
3.7	Mortality by age and duration.	165
3.7.1	Some preliminary ideas.	165
3.7.2	Select tables and ultimate tables.	166
3.7.3	A practical issue.	168
3.8	Mortality dynamics.	168
3.8.1	Mortality trends.	168
3.8.2	Representing mortality dynamics.	171
3.8.3	Probabilities and life expectancy in a dynamic context.	173
3.8.4	Approaches to mortality forecasts.	174
3.8.5	Extrapolation via exponential formulae.	176
3.8.6	Mortality forecasts allowing for random fluctuations.	178

3.9	Moving to a time-continuous context	179
3.9.1	The survival function	180
3.9.2	Other related functions	181
3.9.3	The force of mortality	183
3.9.4	Markers	184
3.9.5	Parametric models	186
3.10	Stochastic mortality	188
3.10.1	Number of people alive in a cohort	188
3.10.2	Deterministic models versus stochastic models	188
3.10.3	Random fluctuations in mortality	191
3.10.4	Systematic deviations in mortality	192
3.10.5	The impact of mortality / longevity risk on life insurance ...	193
3.11	References and suggestions for further reading	194
Life insurance: pricing		197
4.1	Life insurance products	197
4.1.1	General aspects	197
4.1.2	Alterations of a life insurance contract	199
4.2	Discounting cash-flows	200
4.2.1	Premiums, benefits, expenses	200
4.2.2	A lump sum benefit in the case of death	202
4.2.3	A lump sum benefit in the case of survival	202
4.2.4	Combining benefits	202
4.2.5	Actuarial values: terminology and notation	204
4.2.6	Actuarial values: inequalities	207
4.2.7	Actuarial values with zero interest rate	207
4.2.8	The actuarial discount factor	208
4.2.9	Actuarial values: further relations	209
4.2.10	Actuarial values at times following the policy issue	210
4.3	Single premiums	211
4.3.1	The equivalence principle	211
4.3.2	The pure endowment	213
4.3.3	Life annuities	215
4.3.4	The term insurance	217
4.3.5	The whole life insurance	219
4.3.6	Combining survival and death benefits	220
4.3.7	Endowment insurance products	221
4.3.8	The expected profit: a first insight	224
4.4	Periodic premiums	226
4.4.1	An example	226
4.4.2	Level premiums	229
4.4.3	Natural premiums	230
4.4.4	Single premium, natural premiums and level premiums: some relations	233
4.4.5	Single recurrent premiums	234

4.4.6	Some concluding remarks	237
4.5	Loading for expenses	238
4.5.1	Premium components	238
4.5.2	Expenses and loading for expenses	239
4.5.3	The expense-loaded premiums	240
4.6	References and suggestions for further reading	243
 Life insurance: reserving 245		
5.1	Introduction	245
5.2	General aspects	245
5.3	The policy reserve	247
5.3.1	Definition	247
5.3.2	The policy reserve for some insurance products	248
5.3.3	The time profile of the policy reserve	249
5.3.4	Change in the technical basis	256
5.3.5	The reserve at fractional durations	259
5.3.6	The retrospective reserve	263
5.3.7	The actuarial accumulation process	265
5.4	Risk and savings	267
5.4.1	A (rather) general insurance product	267
5.4.2	Recursive equations	268
5.4.3	Risk premium and savings premium	270
5.4.4	Life insurance products versus financial accumulation	274
5.5	Expected profits: a further insight	277
5.5.1	Expected annual profits	277
5.5.2	Splitting the annual profit	278
5.5.3	The expected total profit	280
5.5.4	Cash-flows, profits, premium margins	282
5.5.5	Expected profits according to best-estimate reserving	285
5.6	Reserving for expenses	286
5.7	Surrender values and paid-up values	288
5.8	References and suggestions for further reading	290
 Reserves and profits in a life insurance portfolio 291		
6.1	The portfolio reserve	291
6.1.1	Future portfolio reserves	292
6.1.2	Safe-side reserve versus best-estimate reserve	293
6.1.3	The risk margin	294
6.1.4	The portfolio liability and beyond	297
6.1.5	Risk margin: the "Cost of Capital" approach	298
6.2	The total profit	300
6.2.1	The life fund	300
6.2.2	The expected life fund and the expected total profit	301
r 6.2.3	The total profit: an alternative interpretation	304
6.3	Expected annual profits	305

6.3.1	The expected surplus and the expected annual profits	306
6.3.2	The role of the portfolio reserve	308
6.4	Expected annual profits: a more general setting	312
6.5	References and suggestions for further reading	316
Finance in life insurance: linking benefits to the investment performance		317
7.1	Introduction	317
7.2	Adjusting benefits	319
7.2.1	The general case	319
7.2.2	Addressing specific insurance products	322
7.2.3	Implementing solutions	327
7.2.4	The yield to maturity for the policyholder	331
7.3	Participating policies	335
7.3.1	Participating policies with a guaranteed annual return	336
7.3.2	Participating policies with a guaranteed average return	341
7.4	Unit-linked policies	346
7.4.1	Definition of unit-linked benefits	347
7.4.2	Unit-linked policies without guarantees	348
7.4.3	Unit-linked policies with financial guarantees	353
7.5	Financial options in unit-linked and participating policies	356
7.5.1	The structure of minimum guarantees	356
7.5.2	The valuation of financial options in a unit-linked policy	358
7.6	With-profit policies	360
7.7	Index-linked policies	363
7.8	Universal Life policies	365
7.9	Variable annuities	366
7.10	References and suggestions for further reading	371
Pension plans: technical and financial perspectives		373
8.1	Introduction	373
8.2	Pension programmes	374
8.2.1	Individual and group pension plans	374
8.2.2	Benefits and contributions	376
8.2.3	Timing of the funding	378
8.3	Transferring risks to the provider	379
8.4	Pension savings before retirement	381
8.5	Arranging the post-retirement income	381
8.5.1	Some basic features of life annuities	381
8.5.2	Packaging benefits into the life annuity product	382
8.5.3	Life annuities versus income drawdown	383
8.5.4	Phased retirement	386
8.6	Risks for the provider	388
8.7	"References and suggestions for further reading	392

9 Non-life insurance: pricing and reserving	395
9.1 Introduction	395
9.2 Non-life insurance products	396
9.2.1 General aspects	396
9.2.2 Main categories of non-life insurance products	396
9.3 Loss and claim amount	398
9.4 The equivalence premium	402
9.4.1 The items of the equivalence premium	402
9.4.2 The time-pattern of a claim	402
9.4.3 The expected aggregate claim amount	403
9.5 The net premium	405
9.6 The expense-loaded premium	408
9.7 Statistical data for the equivalence premium	409
9.7.1 Risk premium, claim frequency, loss severity	409
9.7.2 Units of exposure: the case of heterogeneous portfolios	411
9.7.3 Units of exposure: the number of policy years	413
9.7.4 Updating the risk premium to portfolio experience	416
9.8 Stochastic modeling of the aggregate claim amount	420
9.8.1 Modeling the claim frequency	420
9.8.2 Modeling the claim severity	424
9.8.3 Modeling the aggregate claim amount	425
9.9 Risk classification and experience-rating	428
9.9.1 Risk classes and rating classes	428
9.9.2 Risk classification at issue	429
9.9.3 Risk classification at renewal times: individual experience rating	430
9.10 Technical reserves: an introduction	434
9.11 Earned premiums, incurred claim amounts and profit assessment	438
9.12 Deterministic models for claim reserves	441
9.12.1 Run-off triangles	442
9.12.2 The Expected Loss Ratio method	443
9.12.3 The Chain-Ladder method	444
9.12.4 The Bornhuetter-Ferguson method	446
9.12.5 Further aspects	448
9.13 References and suggestions for further reading	449
References	451
Index	455