

DISCRETE MATHEMATICS AND ITS APPLICATIONS

Series Editor KENNETH H. ROSEN

**HANDBOOK OF  
COMBINATORIAL DESIGNS  
SECOND EDITION**

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