

Heinz-Otto Peitgen Hartmutlurgens DietmarSaupe

Chaos and Fractals

Mew Frontiers of Science

With 686 illustrations, 40 in color



Contents

Preface	VII
Authors	XI
Foreword	1
<i>Mitchell J. Feigenbaum</i>	
Introduction: Causality Principle, Deterministic Laws and Chaos	9
1 The Backbone of Fractals: Feedback and the Iterator	15
1.1 The Principle of Feedback	17
1.2 The Multiple Reduction Copy Machine.	23
1.3 Basic Types of Feedback Processes.	27
1.4 The Parable of the Parabola — Or: Don't Trust Your Computer.	37
1.5 Chaos Wipes Out Every Computer.	49
1.6 Program of the Chapter: Graphical Iteration.	60
2 Classical Fractals and Self-Similarity	63
2.1 The Cantor Set.	67
2.2 The Sierpinski Gasket and Carpet	78
2.3 The Pascal Triangle.	82
2.4 The Koch Curve.	89
2.5 Space-Filling Curves.	94
2.6 Fractals and the Problem of Dimension.	106
2.7 The Universality of the Sierpinski Carpet.	112
2.8 Julia Sets.	122
2.9 Pythagorean Trees.	126
2.10 Program of the Chapter: Sierpinski Gasket by Binary Addresses.	132
3 Limits and Self-Similarity	135
3.1 Similarity and Scaling	138
3.2 Geometric Series and the Koch Curve.	147
3.3 Corner the New from Several Sides: Pi and the Square Root of Two.	153
3.4 Fractals as Solution of Equations.	168
3.5 Program of the Chapter: The Koch Curve.	179

4	Length, Area and Dimension: Measuring Complexity and Scaling Properties	183
4.1	Finite and Infinite Length of Spirals.	185
4.2	Measuring Fractal Curves and Power Laws.	192
4.3	Fractal Dimension	202
4.4	The Box-Counting Dimension.	212
4.5	Borderline Fractals: Devil's Staircase and Peano Curve.	220
4.6	Program of the Chapter: The Cantor Set and Devil's Staircase.	226
	Encoding Images by Simple Transformations	229
5.1	The Multiple Reduction Copy Machine Metaphor.	231
5.2	Composing Simple Transformations.	234
5.3	Relatives of the Sierpinski Gasket.	244
5.4	Classical Fractals by IFSs.	252
5.5	Image Encoding by IFSs.	258
5.6	Foundation of IFS: The Contraction Mapping Principle.	263
5.7	Choosing the Right Metric.	274
5.8	Composing Self-Similar Images.	278
5.9	Breaking Self-Similarity and Self-Affinity or, Networking with MRCMs.	283
5.10	Program of the Chapter: Iterating the MRCM.	293
6	The Chaos Game: How Randomness Creates Deterministic Shapes	297
6.1	The Fortune Wheel Reduction Copy Machine.	300
6.2	Addresses: Analysis of the Chaos Game.	307
6.3	Tuning the Fortune Wheel.	321
6.4	Random Number Generator Pitfall.	333
6.5	Adaptive Cut Methods.	341
6.6	Program of the Chapter: Chaos Game for the Fern.	350
7	Recursive Structures: Growing of Fractals and Plants	353
7.1	L-Systems: A Language For Modeling Growth.	357
7.2	Growing Classical Fractals with MRCMs.	364
7.3	Turtle Graphics: Graphical Interpretation of L-Systems.	376
7.4	Growing Classical Fractals with L-Systems.	380
7.5	Growing Fractals with Networked MRCMs.	392
7.6	L-System Trees and Bushes.	397
7.7	Program of the Chapter: L-systems.	402
8	Pascal's Triangle: Cellular Automata and Attractors	407
8.1	Cellular Automata	412
8.2	Binomial Coefficients and Divisibility.	423
8.3	IFS: From Local Divisibility to Global Geometry.	434
8.4	HIFS and Divisibility by Prime Powers.	442
8.5	Catalytic Converters or how many Cells are Black?.	450
8.6	Program of the Chapter: Cellular Automata.	454

	9 Irregular Shapes: Randomness in Fractal Constructions	457
	9.1 Randomizing Deterministic Fractals	459
	9.2 Percolation: Fractals and Fires in Random Forests	463
	9.3 Random Fractals in a Laboratory Experiment	475
	9.4 Simulation of Brownian Motion	481
	9.5 Scaling Laws and Fractional Brownian Motion	491
	9.6 Fractal Landscapes	497
	9.7 Program of the Chapter: Random Midpoint Displacement	503
29	10 Deterministic Chaos: Sensitivity, Mixing, and Periodic Points	507
31	10.1 The Signs of Chaos: Sensitivity	509
34	10.2 The Signs of Chaos: Mixing and Periodic Points	520
44	10.3 Ergodic Orbits and Histograms	525
52	10.4 Paradigm of Chaos: The Kneading of Dough	536
58	10.5 Analysis of Chaos: Sensitivity, Mixing, and Periodic Points	549
63	10.6 Chaos for the Quadratic Iterator	560
74	10.7 Mixing and Dense Periodic Points Imply Sensitivity	569
78	10.8 Numerics of Chaos: Worth the Trouble or Not?	575
83	10.9 Program of the Chapter: Time Series and Error Development	581
93	11 Order and Chaos: Period-Doubling and its Chaotic Mirror	585
97	11.1 The First Step From Order to Chaos: Stable Fixed Points	592
00	11.2 The Next Step From Order to Chaos: The Period Doubling Scenario	603
07	11.3 The Feigenbaum Point: Entrance to Chaos	619
21	11.4 From Chaos to Order: a Mirror Image	628
33	11.5 Intermittency and Crises: The Backdoors to Chaos	640
	11.6 Program of the Chapter: Final State Diagram	651
	12 Strange Attractors: The Locus of Chaos	655
	12.1 A Discrete Dynamical System in Two Dimensions: Henon's Attractor	659
	12.2 Continuous Dynamical Systems: Differential Equations	678
	12.3 The Rossler Attractor	686
	12.4 The Lorenz Attractor	697
	12.5 Quantitative Characterization of Strange Chaotic Attractors: Ljapunov Exponents	709
	12.6 Quantitative Characterization of Strange Chaotic Attractors: Dimensions	721
	12.7 The Reconstruction of Strange Attractors	745
	12.8 Fractal Basin Boundaries	757
	12.9 Program of the Chapter: Rossler Attractor	766
	13 Julia Sets: Fractal Basin Boundaries	769
	13.1 Julia Sets as Basin Boundaries	771
	13.2 Complex Numbers — A Short Introduction	776
	13.3 Complex Square Roots and Quadratic Equations	784
	13.4 Prisoners versus Escapees	789
	13.5 Equipotentials and Field Lines for Julia Sets	800
	13.6 Binary Decomposition, Field Lines and Dynamics	812

13.7	Chaos Game and Self-Similarity for Julia Sets	820
13.8	The Critical Point and Julia Sets as Cantor Sets	826
13.9	Quaternion Julia Sets	837
13.10	Program of the Chapter: Julia Sets	839
14	The Mandelbrot Set: Ordering the Julia Sets	841
14.1	From the Structural Dichotomy to the Binary Decomposition	843
14.2	The Mandelbrot Set — A Road Map for Julia Sets	855
14.3	The Mandelbrot Set as a Table of Content	878
14.4	Program of the Chapter: The Mandelbrot Set	896
A	A Discussion of Fractal Image Compression	903
	<i>Yuval Fisher</i>	
A.1	Self-Similarity in Images	906
A.2	A Special MRCM	908
A.3	Encoding Images	912
A.4	Ways to Partition Images	914
A.5	Implementation Notes	917
B	Multifractal Measures	921
	<i>Carl J. G. Evertsz and Benoit B. Mandelbrot</i>	
B.1	Introduction	922
B.2	The Binomial and Multinomial Measures	927
B.3	Methods for Estimating the Function $f(a)$ from Data	938
B.4	Probabilistic Roots of Multifractals. Role of $\psi(a)$ in Large Deviation-Theory	944
B.5	Some Applications, and Advanced Multifractals	952
	Bibliography	955
	Index	971